<u>A Region External to the Ergosphere with Complex Angles between Coordinate Axes</u>

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In general relativity, the appearance of off-diagonal terms in spacetime metrics indicates non-orthogonal coordinate axes. It is shown that there exists a region outside the ergosphere in such spacetimes where the angle between axes is complex.

In any spacetime with macroscopic movement of mass-energy, off-diagonal terms appear in the metric. Two examples of this are the well-known Kerr metric [1], and a Schwarzschild metric with a Lorentz boost (first described in [2] and first analyzed in [3]). These off-diagonal terms represent a "bending" between coordinate axes such that they are no longer orthogonal. The angle between the axes is easy to determine : using t and φ from the Kerr metric as examples, then by the definitions of the metric tensor g and the dot product

$$g_{t\varphi} = \boldsymbol{e}_t \cdot \boldsymbol{e}_\varphi = e_t\, e_\varphi \cos \alpha$$

where $\boldsymbol{e}_i$ is a coordinate basis vector and α is the angle between the axes. But also by definition

$$g_{tt} = \boldsymbol{e}_t \cdot \boldsymbol{e}_t = e_t e_t$$

so $e_t = g_{tt}^{1/2}$ and likewise $e_\varphi = g_{\varphi\varphi}^{1/2}$, thus the angle between the axes is given by

$$\alpha = \cos^{-1}(g_{t\varphi}(g_{tt}g_{\varphi\varphi})^{-1/2})$$

This angle becomes zero when $g_{t\varphi}(g_{tt}g_{\varphi\varphi})^{-1/2} = 1$. While it may be difficult (if not impossible) to solve this equation analytically for a given metric, it is trivial to find the boundary surface computationally. And it has been found that this surface defines a region *outside* the ergosurface (defined as $g_{tt} = 0$) for which the angle between the axes is complex. Figure 1 shows both the ergosurface (dotted line) and the α = 0 boundary (solid line) of the Kerr metric in the r-θ plane for a body with high angular momentum (the "stair-step" effect is due to the resolution of the computational process).

Figure 2 shows the same results in the x-y plane for a massive object moving at 0.8c in the +x direction (the gaps at y=0 are due to the computational process).

In both cases it can be seen that there is a region outside the ergosurface where $g_{t\varphi}(g_{tt}g_{\varphi\varphi})^{-1/2} > 1$, and thus the angles between the coordinate axes becomes complex. **The nature of the physics of the space-time between this surface and the ergosphere has not yet been determined to the best of my knowledge, and perhaps warrants further investigation.**

<u>References</u>

[1] : M. P. Hobson, G. P. Efstathiou & A. N. Lasenby, *General Relativity: An Introduction for Physicists* (Cambridge University Press, Cambridge, UK, 2006).
[2] : J. B. Hartle, K. S. Thorne, and R. H. Price in *Black Holes: The Membrane Paradigm*,

edited by K. S. Thorne, R. H. Price, D. A. Macdonald (Yale U. Press, New Haven, Conn., 1986), Ch. V.
[3] : F. S. Felber, arXiv:gr-qc/0505098v3, (2009).

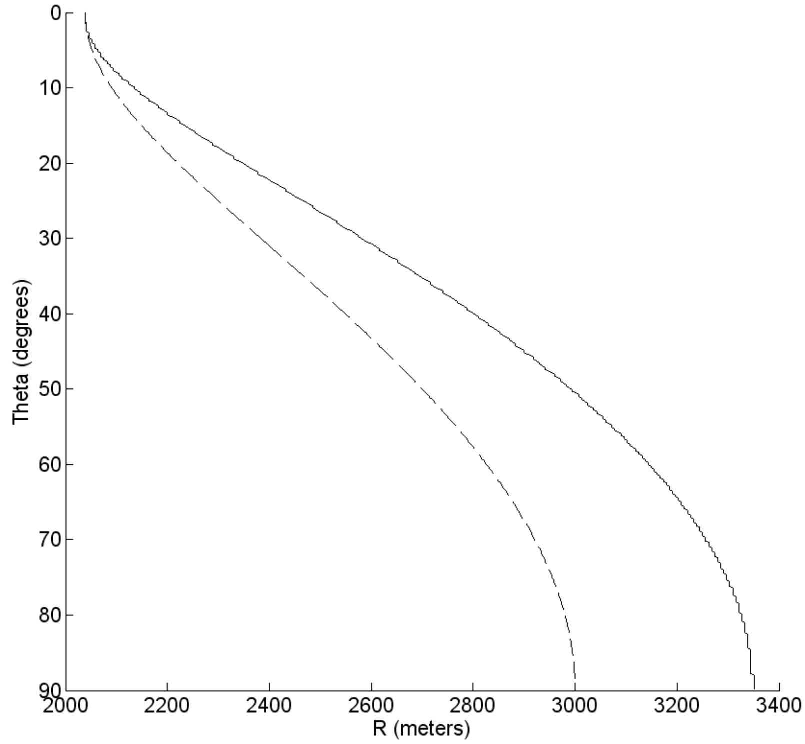

Figure 1 – Ergosurface (dashed) and α=0 surface (solid) for a rotating mass

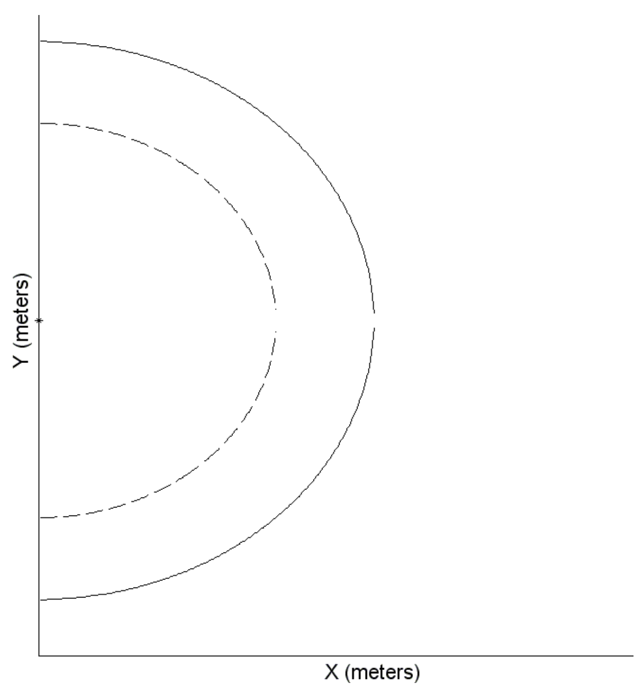

Figure 2 – Ergosurface (dashed) and α=0 surface (solid) for a mass moving at 0.8c in +x direction