The State of the Universe [2010]

"There is only data and the interpretation of data"

(green text = assumptions)

Current thinking in cosmology says that the universe is filled with "dark matter" and "dark energy". What are they, why do we think this, and how do they fit in to general relativity? We'll have to answer these questions in reverse order….

Assuming spacetime is isotropic (same in all directions; proven true), homogenous (same everywhere; implied by isotropy), and spherically symmetrical, we get the Friedmann-Lemaitre-Roberson-Walker metric :

$$ds^2 = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + sin^2\theta d\phi^2) \right]$$

Where

$a(t)$ is the "scale factor", representing how "big" space is at the moment t

$a(t)$ may be "normalized" : $a(t) = R(t) / R_0$ where R_0 is the size right now, so it goes from 0 to 1

$k = -1$: 3-D space is "open" and infinite (like a saddle)

$k = 0$: 3-D space is "flat" and infinite (like a plane)

$k = +1$: 3-D space is "closed" and finite (like a sphere)

So time passes but does not stretch, and space is expanding uniformly in all directions. The coordinates (t, r, θ, φ) are called **co-moving** coordinates, because two objects can remain at their coordinates at all times, while the proper distance between them changes with time according to how the scale factor *a(t)* changes (imagine two dots on a balloon whose coordinates are fixed, while the balloon is blown up). Observations indicate that not only is the universe expanding, but the expansion is accelerating. "Dark energy" will be used here to generically refer to whatever is causing the source of acceleration of the expansion of the universe, altho it goes by several other names ("cosmological constant", "vacuum energy", "quintessence", etc.).

a(t) can be related to the *cosmological* redshift *z* (GR1e) of light from stars : $a(t_e) = 1 / (z+1)$ where t_e is the time the light was emitted. *z* is also related to how long ago something happened, so astronomers often use *z* to represent a time axis. Keep in mind that a bigger *z* means farther into the past.

Modeling the universe as a perfect fluid with $\rho(t)$=mass density and $P(t)$=pressure, we get :

$$T_{\mu\nu} = (\rho + P/c^2) u_\mu u_\nu + g_{\mu\nu} P$$

(note: T is an energy density)

And we must also choose an **equation of state** which describes the kind of mass-energy : $w(t) = P/\rho$

Dust (P=0) : w=0	"matter dominated" (momentum is << mass-energy)
Radiation : w=1/3	"radiation dominated" (black body incoherent radiation → 1/3)
Vacuum : w= –1	"vacuum dominated" (cosmological constant $\Lambda \neq 0$ in Einstein's equation)
	other models of dark energy have $-1 < w < 0$

"Dominated" means we pretend that that particular form of mass-energy is the only one in the universe.

Using comoving coordinates that expand with the universe, Einstein's equation (including the "cosmological constant" Λ) results in two partial differential equations :

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{\Lambda c^2}{3}$$

- The Hubble parameter is defined as $H(t) = \dot{a} / a$, and its units are usually given in (km/sec)/Mpc

- Currently the value is believed to be about H=72±2 (5 recent measurements, independent of Ω below)
- *h*(t) is a "unitless" Hubble constant = H/100; the best single value to date [2010] is h = 0.704

The pressure term in the second equation is ignored in all of the following because dividing it by c^2 makes its contribution much smaller than ρ. The relative motions of galaxies within the local group has a "pressure" ≈ 10^{-17} N/m^2 whereas $\rho \approx 10^{-27}$ kg/m^3, but $3P/c^2 \approx 10^{-33}$ kg/m^3, a million times smaller than ρ!

Starting from the first equation :

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

And dividing by H^2 :

$$1 = \frac{8\pi G}{3H^2}\rho - \frac{kc^2}{a^2H^2} + \frac{\Lambda c^2}{3H^2}$$

We then define

$$1 = \Omega_{ME} - \Omega_C + \Omega_\Lambda$$

Where Ω_{ME} is the ratio of the actual mass-energy density ρ to the "critical density" $\rho_c(t)$, which is the value needed for the universe to be flat (k=0). Today $\rho_c \approx 10^{-26}$ kg/m^3, which is 5-6 hydrogen atoms per cubic meter of space. Ω_Λ is the dark energy component, and Ω_C is called the "curvature" component. Assuming that the observable universe is the entire universe (which is not necessarily so), $\Omega_C \approx 0.1$k. However, it appears that the spatial universe is flat (see below), so k=0 and this term vanishes. Thus we are left with $\Omega_{ME} + \Omega_\Lambda$, which we define as Ω (sometimes called "curvature"). Values of k (above) correspond to values of Ω as follows :

$\Omega < 1 \leftrightarrow k = -1$ (saddle)
$\Omega = 1 \leftrightarrow k = 0$ (flat)
$\Omega > 1 \leftrightarrow k = +1$ (sphere)

Separating the matter and energy components : $\Omega(t) = \Omega_M(t) + \Omega_\gamma(t) + \Omega_\Lambda(t)$

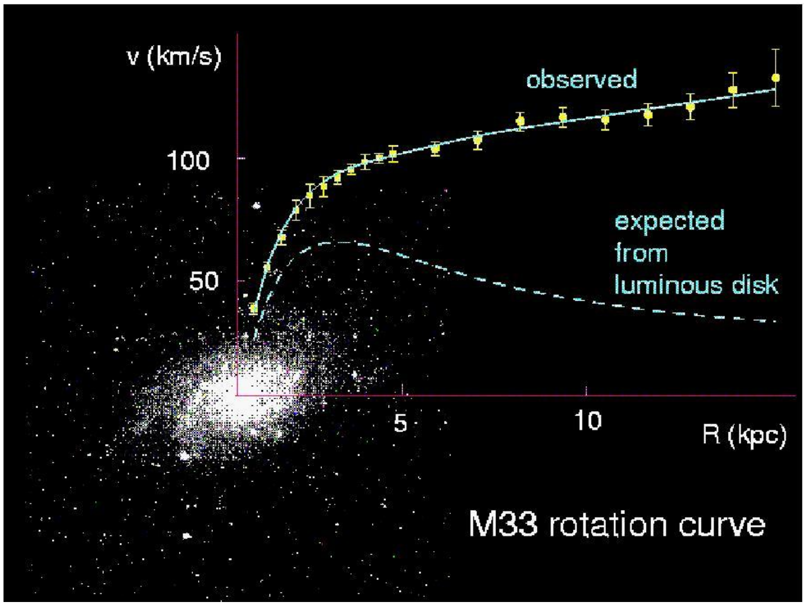
Ω_M = all physical matter = Ω_b (baryons+electrons) + Ω_{cdm} (cold dark matter) + Ω_n (neutrinos)
Ω_γ = radiation and "relativistic" particles moving close to the speed of light (also called Ω_r)
Ω_Λ = dark energy

The reasons for needing the Ω_{cdm} and Ω_Λ terms will be explained below. Note that neutrinos could be lumped with Ω_M because they are physical matter, but are usually included in Ω_γ because their velocity is probably close to *c*, so they are relativistic. Also, all matter in the very early universe would have been relativistic.

Ω_k is defined as 1 – Ω and represents the difference from flat (and to account for unknown sources).

The best estimates of Ω_γ and Ω_n to date [2010] are $\Omega_\gamma = 2.5\text{x}10^{-5}$ (for just photons) and $\Omega_n = 10^{-5}$ to 10^{-3}, so they are usually ignored. Up until 1998, it was thought that (essentially) $\Omega_M = 1$ and $\Omega_\Lambda = 0$.

The main evidence for dark matter comes from the rotational speed of stars in galaxies. When we look at galaxies, we can measure how quickly they rotate in the central part, the intermediate parts, and the outskirts. If there were only stars and gas (normal matter), we'd expect the outskirts to rotate more slowly than the inner parts, the same way that the outer planets orbit our Sun more slowly than the inner ones. But they don't – the outskirts move *at the same speed* as the inner portion. The difference between the expected and the observed is easy to see :



The easiest way to account for the difference is to assume that there is up to **10 times** more matter surrounding the galaxies than what we can see. Since it doesn't radiate, it is "dark". Sometimes it is called "cold" dark matter, meaning it doesn't have a lot of kinetic energy ($v << c$).

What this dark matter could be is more difficult to pin down – it may be a new kind of neutrino, or the lightest-weight "supersymmetric" particle (a hypothetical particle whose existence has not been confirmed), or any number of other things. It was recently discovered [2011] that there are 3 times more red dwarf stars in the universe than was previously thought, and huge volumes of hydrogen in-between the galaxies. While these do not solve the problem, it just goes to show that there are lots of things about the universe we don't know yet.

It's also possible that dark matter doesn't really exist because we are interpreting the physics incorrectly (which means the "expected" curve in the above figure is wrong). For example, perhaps we are not modeling the mass distribution in the galaxy accurately enough, or the angular momentum from the rotation of stars and the rotation of the galaxy adds significant frame dragging, which is currently ignored. It's also possible that the equations change at small accelerations (MOND) or in some other way we're not aware of.

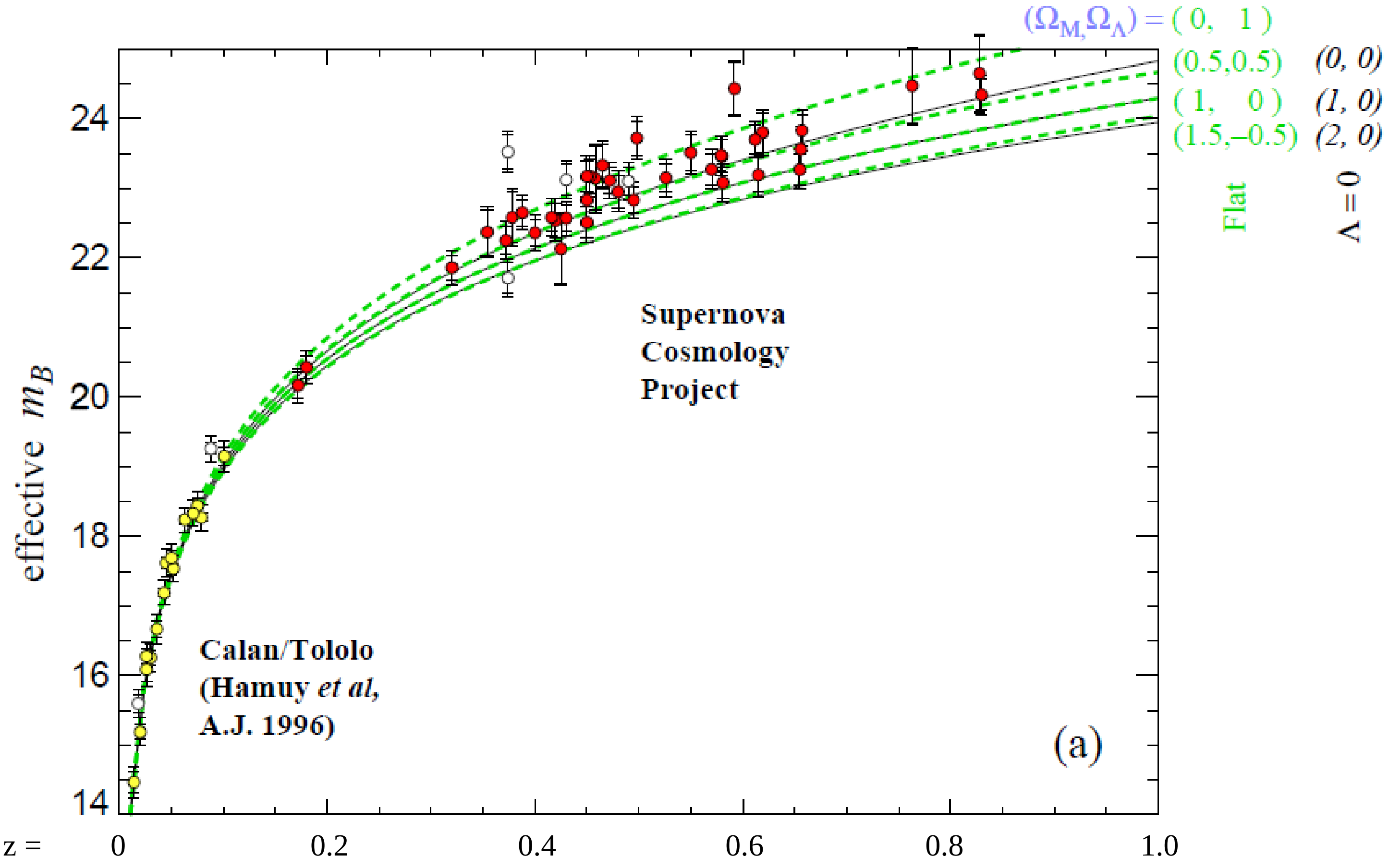
It became apparent as early as 1930 that the universe was expanding, but it wasn't until 1998 that data indicated that the expansion of the universe was accelerating. This was surprising, because in a universe filled with proton, neutrons, electrons, and photons (all of which only attract), the expansion could only be decelerating.

The evidence for an accelerating expansion currently comes from many different sources, including gravitational lensing and x-ray clusters, but the first indication came from the redshifts of a particular kind of supernova (SNe) called "1a". Because all 1a supernovas explode in the same way, it is easy to determine how far away they are. And when we compare that independently-determined distance with their measured redshift, they appear to be farther away than their redshift would indicate. Since we trust the supernova-estimated distance, the lower-than-expected redshift means that the universe was expanding at a slower rate in the distant past than it is now. Conversely, that means it is expanding at a faster rate (accelerating) now.

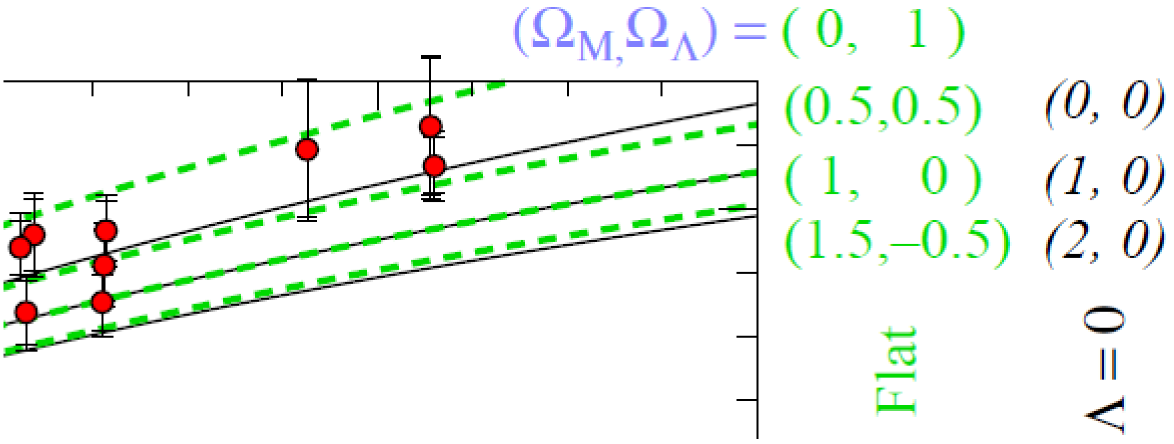
If "dark energy" is really the Λ in Einstein's equation, it is called the "cosmological constant", and then $w = -1$ over all time. It could also be the "vacuum energy" of quantum mechanics (related to the Casmir effect), which would act the same as Λ (and be constant over time), but the measured value of Λ is **much** smaller than the vacuum energy predicted by quantum mechanics. Or, if a hypothetical scalar field called "quintessence" is the source of the acceleration, then $\Lambda=0$ and another term enters the equation. In this case, "Ω_Λ" becomes a function of time, as does *w* which can vary from 0 to –1.

Again, it's also possible that dark energy doesn't exist, due to a variety of possible reasons : the formulas might contain too many simplifications (perfect fluid), which means that relativistic aspects are being ignored (such as the angular momentum of galaxies). Perhaps *our particular region* of spacetime is not perfectly isotropic and homogenous (our first assumptions), or Einstein's equation is not correct over cosmological distance and time scales. Or maybe the various "constants" in the equation (G, c, etc.) vary over cosmological time scales. A very real possibility is that torsion, which was ignored by Einstein, may account for the observed effects.

Since pressure, Ω_γ and Ω_n are so small, they are ignored. This leaves $\Omega_M = \Omega_b + \Omega_{cdm}$ as the major source of decelerating the expansion, and Ω_Λ as the only source of accelerating it (Ω_C does not affect the acceleration of expansion). The values of these two parameters have been estimated from various sources of astronomical data. Looking at the supernova data that had been accumulated by 1999 :

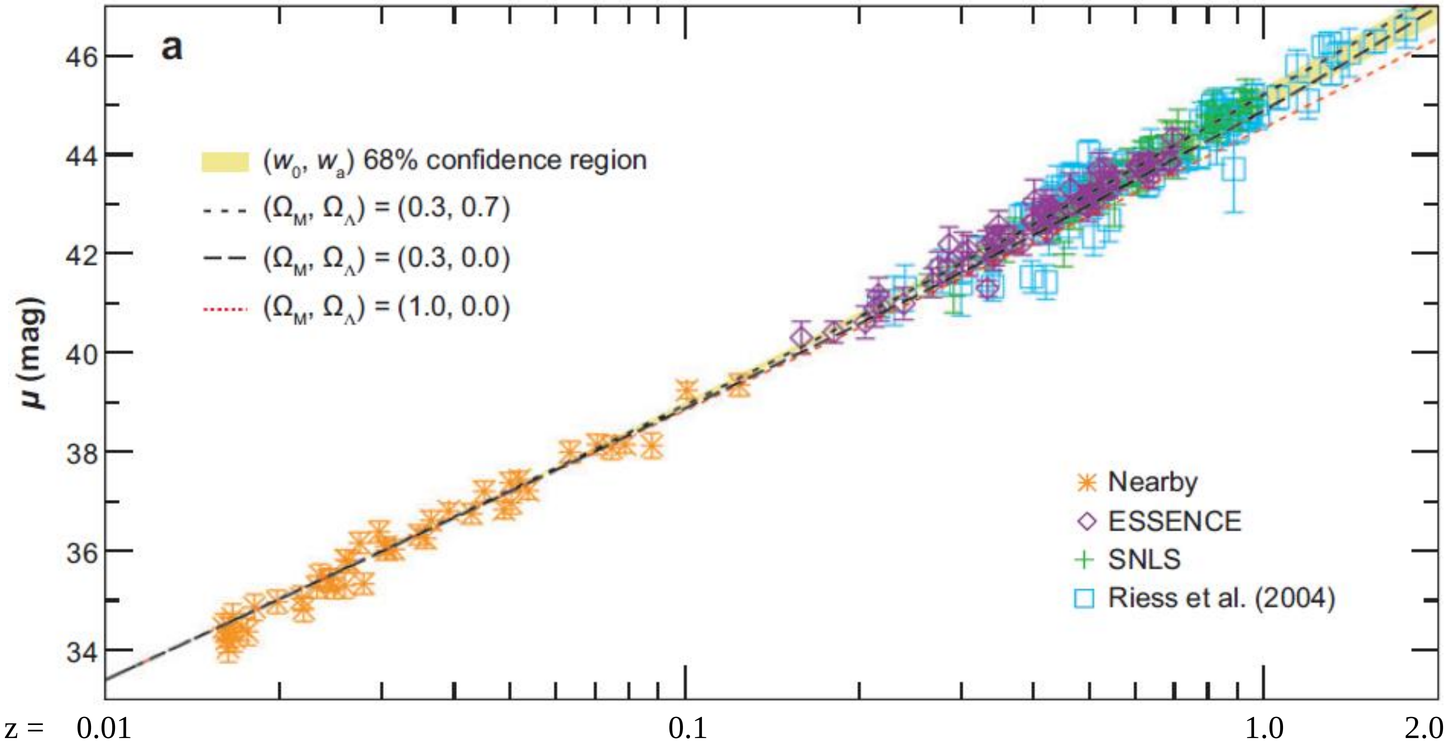


And zooming in on the top right corner :

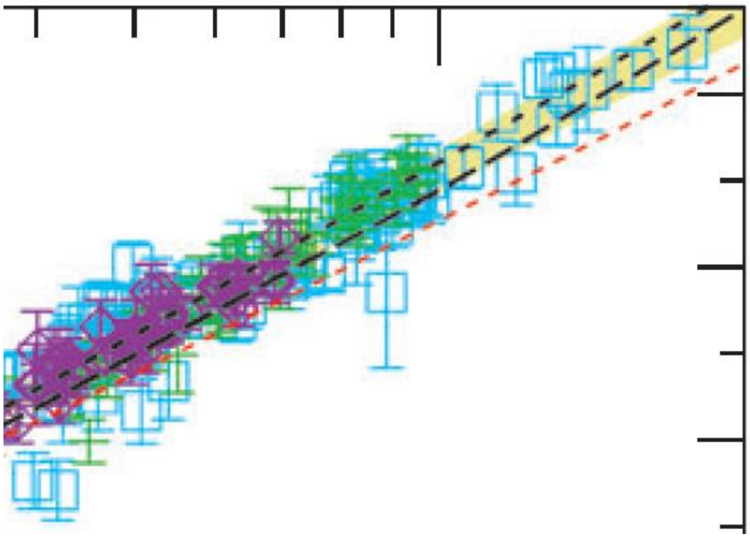


The 4 green dotted lines and the 3 black lines represent 6 different combinations ("models") of Ω_M and Ω_Λ. While it is hard to tell by eye which line fits the data best, data analysis techniques can tell that the combination of $\Omega_M = 0.28$ and $\Omega_\Lambda = 0.72$ works best, **assuming a flat universe**. This was the first indication that the expansion was accelerating.

A few years later, after much more data had been gathered (same information, different graph format) :

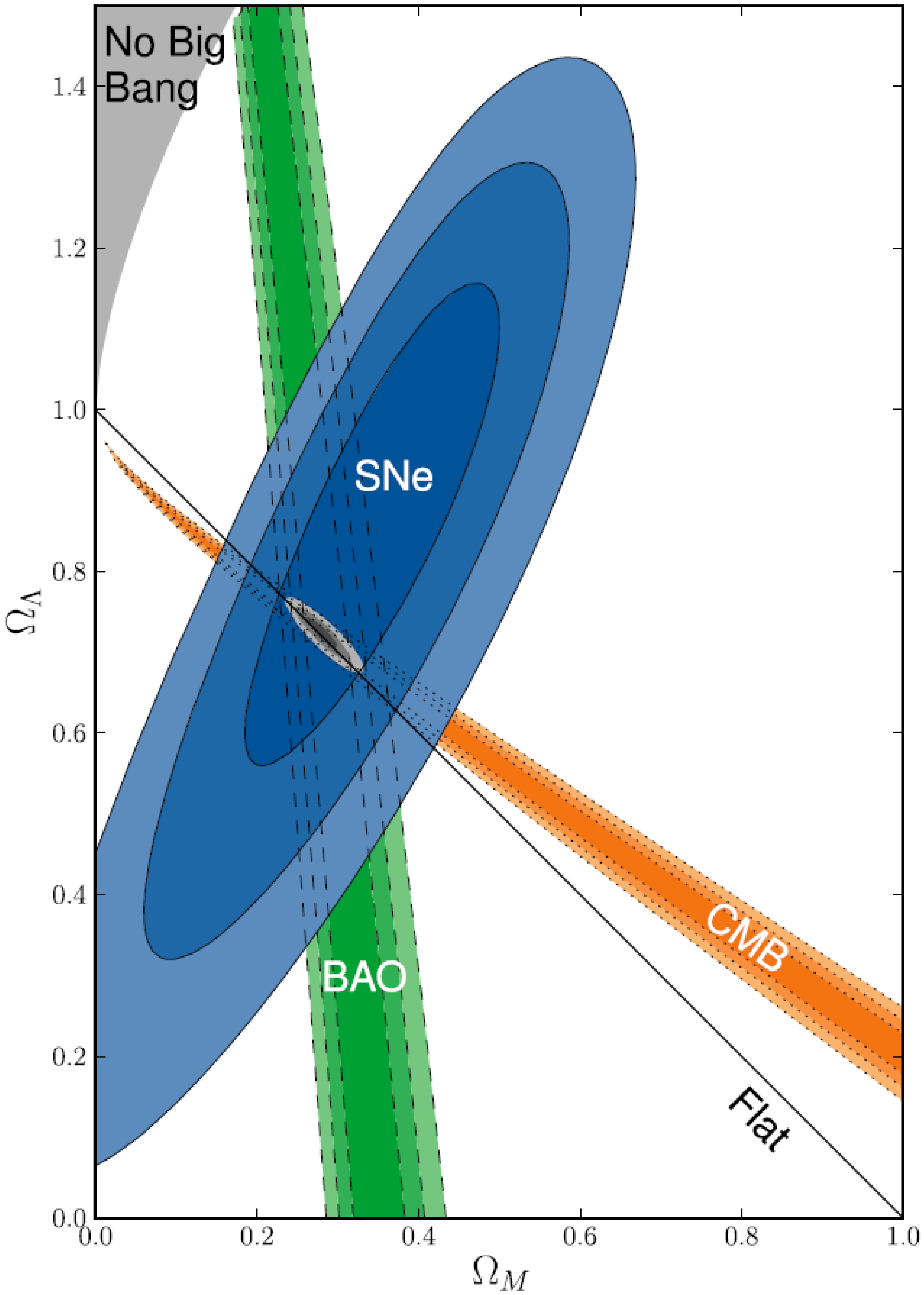


Again, zooming in on the upper right corner, it's still hard to distinguish between the models :



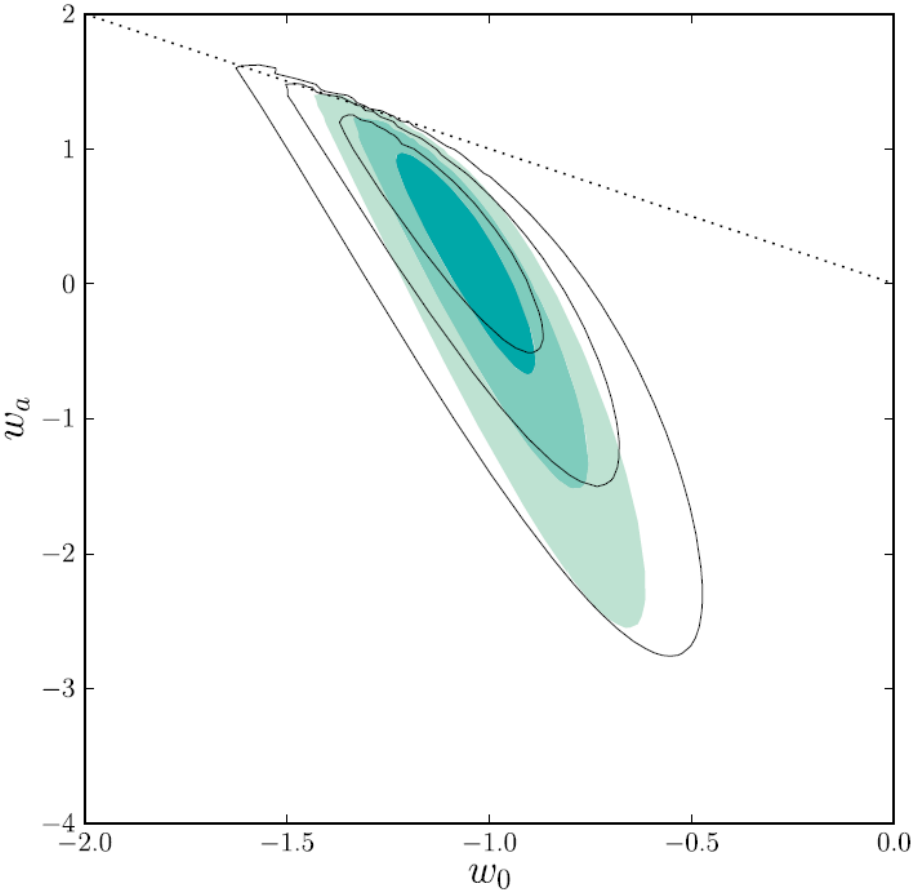
It is important to understand that data analysis at this level is very tricky! It is very difficult to process the data to remove factors that don't contribute to what we want to look at (such as the *gravitational* redshift as the light leaves the gravity well of the supernova), or to correct for factors that may change the results (such as the light from the supernova travelling thru galactic dust). And there are both systematic errors (equipment limitations) and statistical errors that will always be present. In addition, dark energy was defined by the brightness of the most distant supernova, where they become exponentially fainter and the data becomes much less accurate.

Fortunately, we can combine information from multiple sources to get better estimates. Note that in any Ω_M/Ω_Λ graph, below&left of the “flat” line the universe would be open, and above&right of it the universe would be closed. Also, if $\Omega_\Lambda > 0$ the universe expands forever, and if $\Omega_\Lambda < 0$ it would eventually collapse. The blue “SNe” ovals are from the above supernova data. The following figure [2008] **assumes w = –1**.



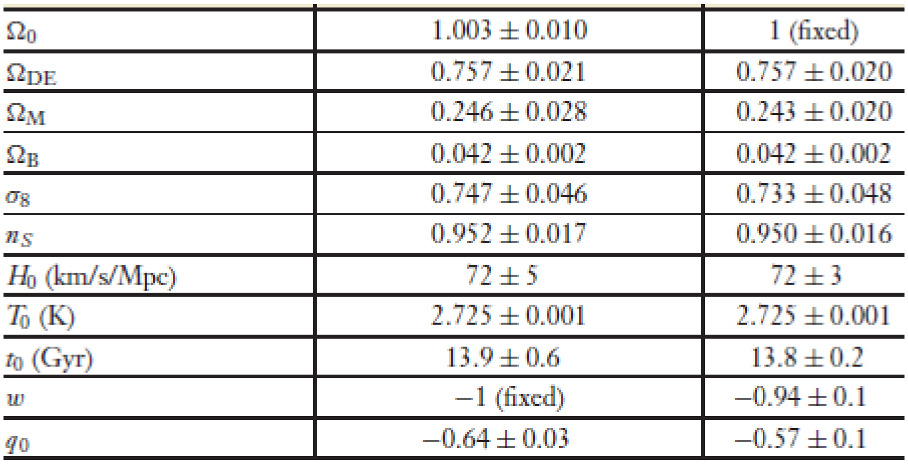
The best estimate for this data is $\Omega_M = 0.27$ and $\Omega_\Lambda = 0.73$ [2008], but as can be seen there is still a large amount of uncertainty in both values (the size of the gray ovals)!

As mentioned before, there could be many different reasons for the acceleration, and they would show up in the equations in different ways, and with different values of *w*, which might change over time. The current approach to test different models is to make *w* a function of two parameters : $w = w_0 + w_a z/(1+z)$, then find the best values of w_0 and w_a that fit the data. w_0 is the value of *w* now, and w_a describes how the dark energy changes with time. As can be seen in the figure below (**which assumes a flat universe**), $w_0 = -1$ and $w_a = 0$ are within the region of best fit (indicating that the acceleration is acting like the cosmological constant Λ in Einstein's equation), but again there is lots of room for other acceptable values. The best estimate from the data that created the figure is w = –0.997 ± 0.08 [2010].



It should be apparent by now that there are *several* parameters (Ω_b, Ω_{cdm}, Ω_Λ, w_0, w_a, and many others not mentioned here) that *all* need to be determined from *multiple* sources of data. The values they take to best fit the data depend on which datasets are used (which change year-by-year as new data is gathered), and what initial assumptions are made (like w = –1 or k = 0 in the previous two figures). For example, if the assumption of a flat universe is removed in the previous figure, the best estimate becomes w = –1.04 ± 0.09 [year].

In the following table [2006], the first column assumes dark energy acts like Λ (w = –1) and the second assumes a flat universe (k = 0 ↔ Ω = 1). It can be seen that not only do the other parameters arrive at pretty much the same values regardless of the assumptions, but that w ≈ –1 and Ω ≈ 1 when they are not fixed, all of which are good signs that the data is self-consistent, and the estimated values are close to the actual values.



Ω_0	1.003 ± 0.010	1 (fixed)
Ω_{DE}	0.757 ± 0.021	0.757 ± 0.020
Ω_M	0.246 ± 0.028	0.243 ± 0.020
Ω_B	0.042 ± 0.002	0.042 ± 0.002
σ_8	0.747 ± 0.046	0.733 ± 0.048
n_S	0.952 ± 0.017	0.950 ± 0.016
H_0 (km/s/Mpc)	72 ± 5	72 ± 3
T_0 (K)	2.725 ± 0.001	2.725 ± 0.001
t_0 (Gyr)	13.9 ± 0.6	13.8 ± 0.2
w	-1 (fixed)	-0.94 ± 0.1
q_0	-0.64 ± 0.03	-0.57 ± 0.1

We can even make no assumptions, and allow Ω and *w* to also be "free" parameters, in which case the best values come out to be $w = -1.08 \pm 0.12$ and $\Omega = 1.03 \pm 0.016$ [2007]. Note that in this case, $\Omega=1$ is not an allowable value!

The best estimates to date [2010] that I could find are :

$\Omega_\gamma = 2.5\text{x}10^{-5}$
$\Omega_n = 10^{-5}$ to 10^{-3}
$\Omega_b = 0.045 \pm 3\%$ Ordinary matter (stars, planets) is only 4.5% of the universe's mass-energy
$\Omega_{cdm} = 0.22 \pm 3\%$ "Dark matter" is 22%, about 5 times as much!
$\Omega_M = 0.27 \pm 3\%$
$\Omega_\Lambda = 0.73 \pm 2\%$ "Dark energy" comprises 73% of the mass-energy of the universe!
$\Omega_k = -0.006 \pm 0.008$ The universe is very close to spatially flat ($\Omega_k = 0$ so $\Omega = 1$)
$w_0 = -0.997 \pm 0.08$ \ "Dark energy" is best represented
$w_a = 0$ / by the cosmological constant Λ

So some authors say we are living in a flat, "matter dominated" universe, where $\rho_{matter}/\rho_{radiation} \approx 10^4$. On the other hand, $\Omega_\Lambda/\Omega_M=2.7$ and $w \approx -1$, so it appears that the universe is currently "vacuum dominated".

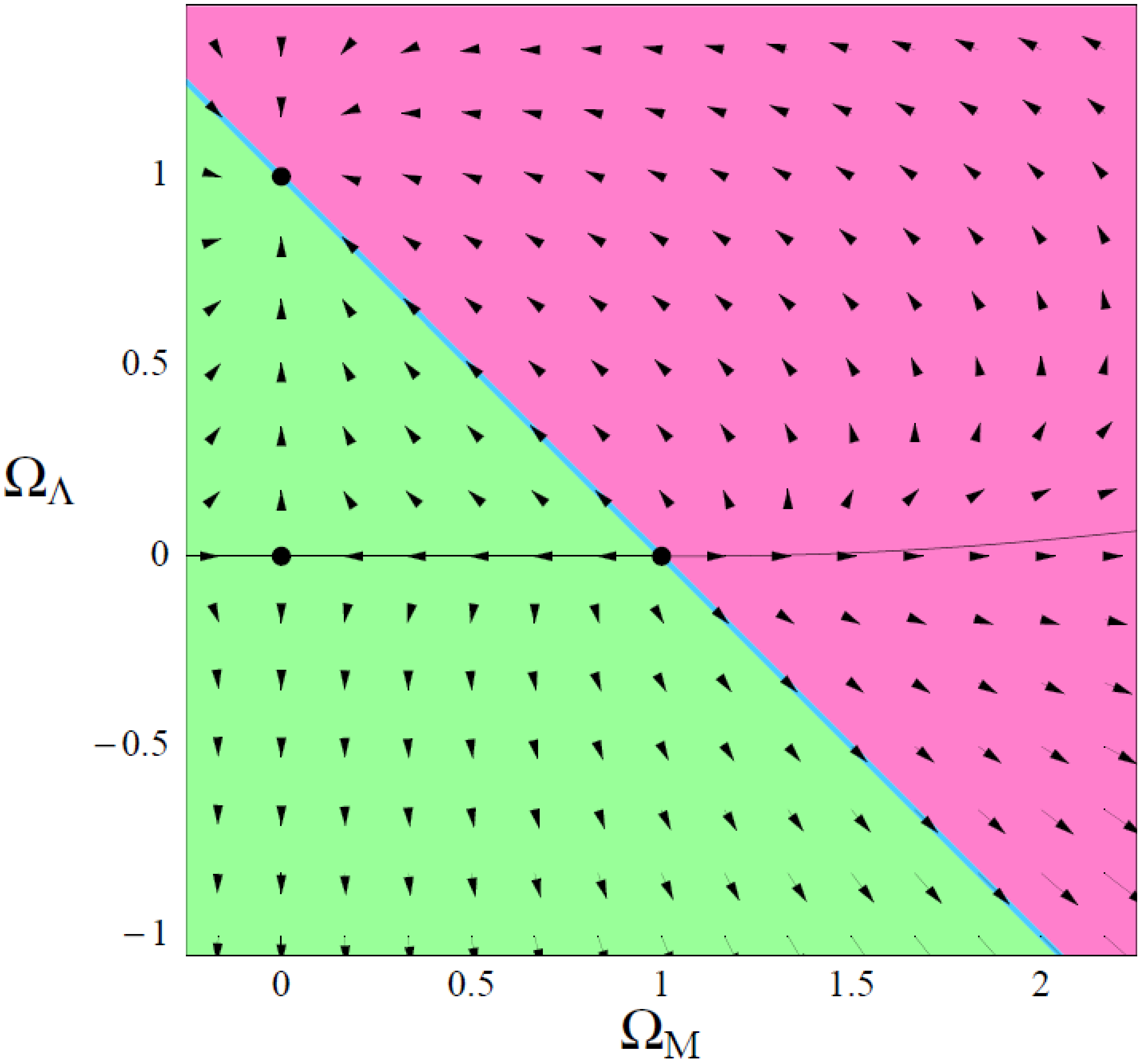
While ±3% accuracy may not sound bad, keep in mind that most physical constants (c, G, etc.) are known to within ±0.01% to 0.0000001%! In particular, if w_0 and Ω are not exactly –1 and 1 respectively, that would make a *huge* difference in our understanding of the universe! For example, if the true value of Ω is 1.001 (which, among other things, means that $k=+1$ and $\Omega_C \approx 0.1$ does **not** vanish), we would not know that until the %error is less than ±0.05%. So the bottom line is, we really don't know the state of the universe!

Since we don't even know what dark matter and dark energy might be (if indeed they really exist), what do we know *for sure*?

The universe is expanding
The universe is very close to flat

That's it!

The values of Ω_M and Ω_Λ also determine the future of the universe :

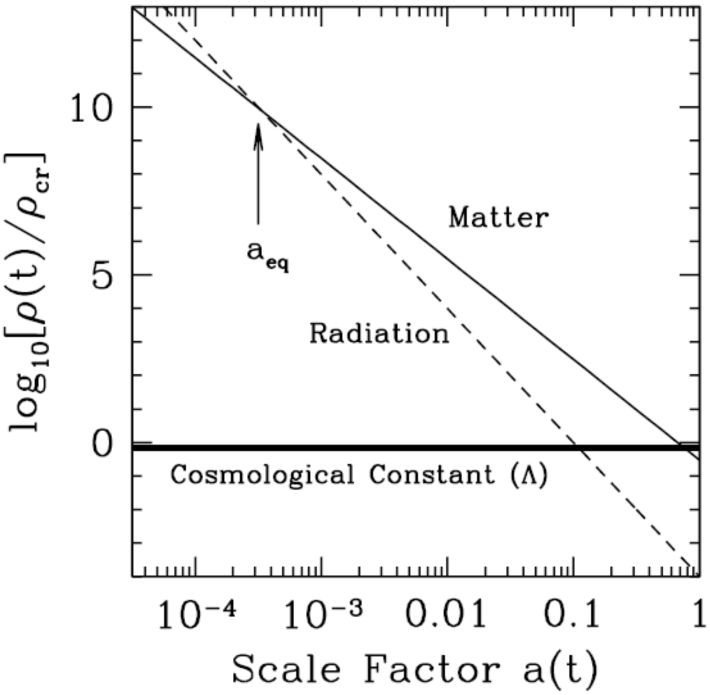


This figure includes three fixed points, at (Ω_M,Ω_Λ) equal to (0,0), (0,1), and (1,0). The attractor at (0,1) is known as de Sitter space – a universe with zero matter density, dominated by a cosmological constant, and where a(t) grows exponentially with time. The fact that this point is an attractor on the diagram is another way of understanding the cosmological constant problem : a universe with initial conditions located at a generic point on the diagram will eventually end up at (0,1) if it began above the recollapse line (the black line going from (0,0) thru (1,0) to the right), and flow to $\Omega_M = \infty$ (collapse, "Big Crunch") if it began below that line. Since our universe has expanded by many orders of magnitude since early times, it must have begun at a fixed point in order not to have evolved either to de Sitter space or to a Big Crunch. The only other two fixed points on the diagram are the saddle point at (0,0) corresponding to an empty universe, and the repulsive point at (1,0) known as the Einstein–de Sitter solution. Since our universe is not empty, the favored solution from this combination of theoretical and empirical arguments is the Einstein–de Sitter universe : (Ω_M,Ω_Λ) = (1, 0). Inflation provides a mechanism whereby the universe can be driven to the line $\Omega = \Omega_M + \Omega_\Lambda = 1$ (blue diagonal line = spatial flatness), so Einstein–de Sitter is a natural expectation if we imagine that some unknown mechanism sets Ω_Λ = 0. But, the current interpretation of the data put us on the blue line at (Ω_M,Ω_Λ) = (0.27, 0.73). Oops!

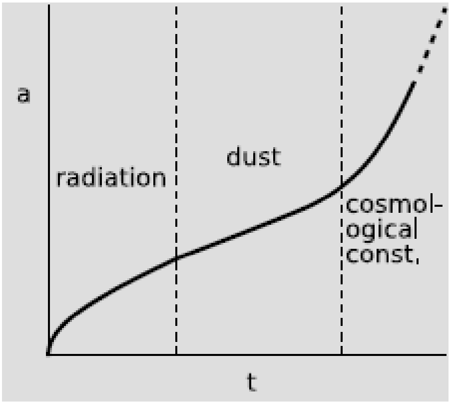
Assuming ***w* is constant** and the **universe is spatially flat**, then we can determine how the scale factor varies with time, and how the various mass-energy densities vary with the radius of space (= scale factor) :

Mass-Energy Type	**w = P / ρ**	**a(t)**	**ρ(a)**
Radiation	1/3	$\alpha\ t^{-1/2}$	$\alpha\ a^{-4}$
Matter	0	$\alpha\ t^{-3/2}$	$\alpha\ a^{-3}$
“Curvature”	–1/3	$\alpha\ t$	$\alpha\ a^{-2}$
Cosmol.Constant	–1	$\alpha\ e^{t}$	constant

For example, the density of matter is proportional to $a^{-3} = 1/r^3 = 1/m^3$ is the standard definition of density. Because radiation varies as $1/r^4$, radiation was the dominant factor when the universe was very small. As the universe expanded, matter became more important, and since the “density” of the cosmological constant doesn’t vary, as the universe continues to expand it eventually <u>will</u> become the most important factor :

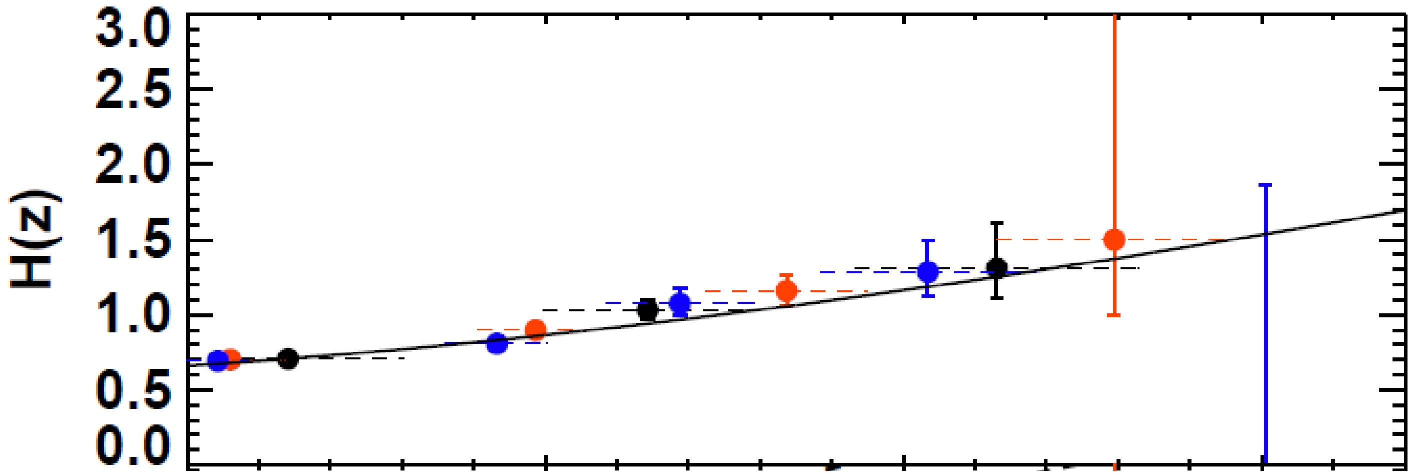


Thus the universe has gone thru three distinct eras : when radiation density was largest at very early times, the universe was “radiation dominated”. As the universe expanded, matter (“dust”) density was highest, and the universe was “matter dominated”. As the expansion continued, eventually Λ became largest, and the universe is just now beginning to become “vacuum dominated” by the cosmological constant. The following figure shows how the size of the universe changed during these eras :



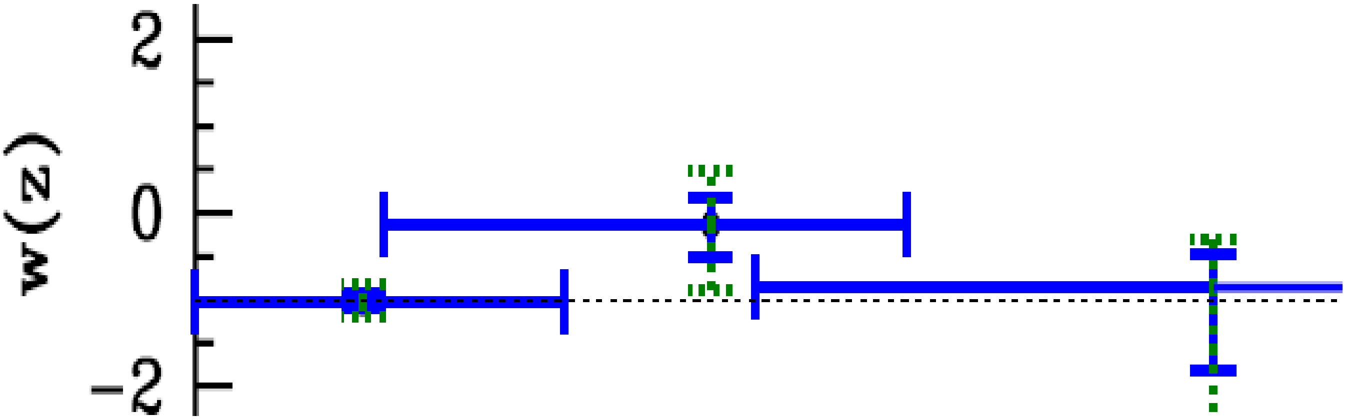
a α $t^{-1/2}$ $t^{-3/2}$ e^{t}

Finally, there is no reason to assume that the values of these parameters are exactly constant over the entire age of the universe, which spans dozens of orders of magnitude in size (since inflation ended) and mass-energy density. The following figure shows the possible evolution of H over time [2008] :



z =	0	0.5	1.0	1.5
~10^9yrs ago	0	5	7.7	9.3

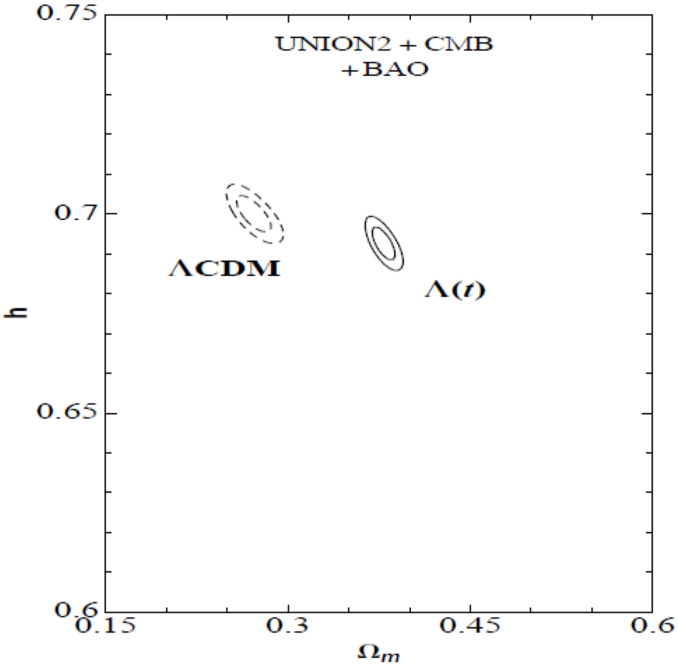
It is also possible that *w* has changed over the eons, which would make sense as the universe went from radiation-dominated (*w* = 1/3) to matter-dominated (*w* = 0) to vacuum-dominated (*w* = –1) [2008] :



z =	0	0.5	1.0	1.5
~10^9yrs ago	0	5	7.7	9.3

Altho note that the estimated value for *w* in the distant past *does not match the expected radiation-dominated value of 1/3.*

Interestingly, there is evidence that Λ is not changing over time, as these results show [2010]. Given the current best estimates of $\Omega_M = 0.27$ and h=0.704, the Λ(t) model is ruled out, giving further support to the idea that dark energy is the cosmological constant in Einstein's equation.



Finally, the Ricci scalar for a FLRW universe is :

$$R = 6\,(\dot{H}/c^2 + 2H^2/c^2 + k/a^2)$$

The value of the Hubble constant now is $H \approx 70.4$ (km/sec)/Mpc $= 2.3\text{x}10^{-18}$ sec^{-1}

And from the figure above showing how H might change over time :

$H \approx 155$ (km/sec)/Mpc about 9.3 billion years ago
$= 5.0\text{x}10^{-18}$ sec^{-1} about $2.9\text{x}10^{17}$ sec ago

So

$\dot{H} = \Delta H/\Delta t \approx -9.3\text{x}10^{-36}$ sec^{-2} (negative because H is getting smaller with time)

Assuming k = 0 :

$$R \approx 6\,(-9.3\text{x}10^{-36} + 1.1\text{x}10^{-35})/c^2 \approx 10^{-52}\ \text{m}^{-2}$$

While this value might be off by as much as a factor of 100 either way, regardless of the actual value a non-zero Ricci scalar means there is some curvature to spacetime, which since we assumed space is flat (k = 0) means either **time is stretching or k ≠ 0 !!!**